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SIMULATION AND ROBUST CONTROL PEAK GYROSCOPE WITH PARAMETRIC UNCERTAINTY

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Abstract. The technology of synthesis of robust control is considered based on the method of "high gain". The method of the Lyapunov function is used for the synthesis of robust control. The nonlinear object with parametric uncertainty is investigated. As an example, a parametric pendulum (peak gyroscope) is considered. The parameters of the object - the frequency of oscillations and the attenuation coefficient during the operation of the system make significant deviations from the nominal. Robust system modeling is done on Simulink. High tracking accuracy is obtained relative to the sliding mode at an oscillatory setting.

Keywords: Uncertainty, robust system, high gain, Lyapunov function, peak gyroscope.
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1 Introduction

As constructive methods for control of the uncertain objects, adaptive and robust systems can be emphasized (Furtat, 2012). Adaptive methods have exhausted their relevance due to the complication of technologies. A distinctive feature of robust systems is the maintenance of the quality indicators of the system when changing the details of the object in a wide range. Moreover, the control device (regulator) is not subjected to structural and parametric adjustment.

In the paper for the synthesis of robust control, as the most adequate, the "high gain" method is used. This method is based on the increasing the open loop gain without violating the stability of the system (Rustamov, 2015, 2018, 2020, 2021).

2 Problem Statement

Let us consider a parametric indefinite object – the parametric pendulum whose parameters vary over a wide range. This oscillator belongs to open oscillatory systems. Changes in parameters, for example, the parameters ω or β lead to the change in the dynamics of the system. This type of pendulum is used to stabilize the angular movement of moving objects.

Consider a peak gyroscope described by the equation (Levant, 2007)

$$\ddot{y} = -a\frac{\dot{R}}{R}\dot{y} - g\frac{1}{R}\sin(y) + \frac{1}{mR^2}u + n(t), \tag{1}$$

Here *y(t)* is the angular displacement, u is the control signal, *n(t)* is the high-frequency noise; $m = 1kg,\ \ g = 9.81m/s^2,\ \ a = 2;\ 0 < R_m \le R \le R_M.$

The initial condition is given as

$$\boldsymbol{y}(0) = (0,\ 0)^T.$$

Load movement equation is

$$R = 0.8 + 0.1\sin(8t) + 0.3\cos(4t).$$

Consider the problem of synthesis of the control u that ensures the motion of the angular displacement of the parametric pendulum *y(t)* along a etalon trajectory $y_d(t)$ with a periodic change in the parameters and load.

Let the etalon trajectory is given as

$$y_d = 0.5\sin(0.5t) + 0.5\cos(t)$$

with the initial condition $y_d(0) = (0.5; 0.25)^T$.

The main components of the model are

$$f(\boldsymbol{y}, t) = -2\frac{\dot{R}}{R}\dot{y} - g\frac{1}{R}\sin(y) + n(t), \quad b(\boldsymbol{y}, t) = \frac{1}{mR^2}.$$

Figure 1a and 1b, respectively, shows the kinematic diagram of the pendulum and graphs of the change in the coefficients of the model.

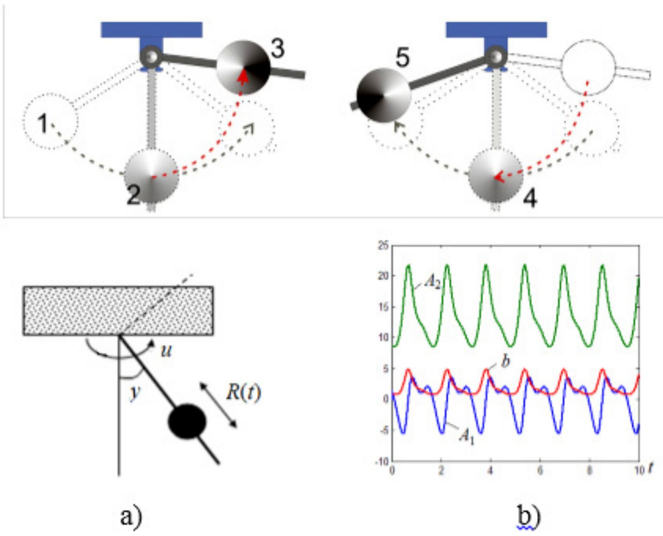

Figure 1: Kinematic scheme and graphs of change model coefficients

In Fig. 1 are taken $A_1 = 2\dot{R}/R, A_2 = g/R, \quad b = 1/mR^2$.

Required quality indicators: overshoot is $\sigma = 0\%$ admissible tracking error is $\delta = \pm 2\%$ settling time is $t_s = 1\ s$.

Since the order of the system is $n = 2$, then, according to Rustamov (2015), the robust controller has two settings. Under the initial condition $\boldsymbol{e}(0) = y_d(0) - y(0) = (0.5;\ 0.25)^T$ the control equation is

$$u = k(3.22e + \dot{e}).$$

Here k is a sufficiently large number (open loop gain); c=3.22 is the switching line slope.

Figure 2a and 2b, respectively, at nominal values $a = 2$, $m=1$ show the dependence of the tracking error $e(t)$ on the gain k and the phase portrait of the system at a fairly large value of $k=120$.

As can be seen from the figures, at $k=120$, the specified quality indicators $\sigma = 0\%, \quad t_s = 1s$ are fulfilled

The case of parametric uncertainty. Let the coefficient a at $\dot{y}(t)$ and the mass of the load m have a deviation $\approx \pm 50\%$ from the nominal one $a = [1;\ 1.5;\ 2;\ \ 2.5]$, $m = [0.5;\ 1;\ 1.5;\ 2]$.

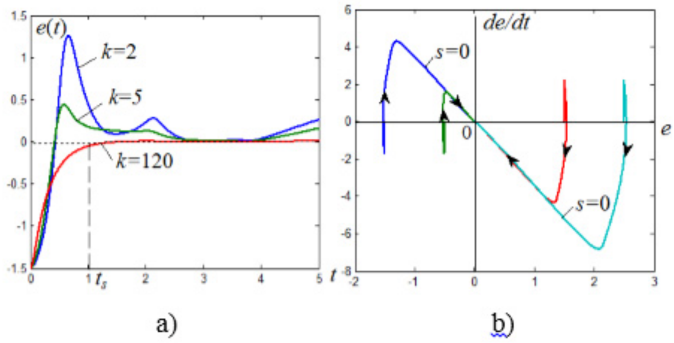

Figure 2: Tracking error and phase portrait of the robust system

Figure 3a, 3b and 3c at k=*120* and the initial condition $\boldsymbol{y}(0) = (2;0)^T$, respectively, show the beam of transient responses $\{y(t)\}$, for 4 combinations of parameters and the corresponding error $\{e(t)\}$ and control $\{u(t\ )\}$.

As can be seen, the density of the beams $\{y(t)\}$ and $\{\ e(t)\}$ is quite high. After $t_s \approx 1$ *c* the bunch of controlled variables $y(t)$ starts tracking the reference trajectory with high accuracy.

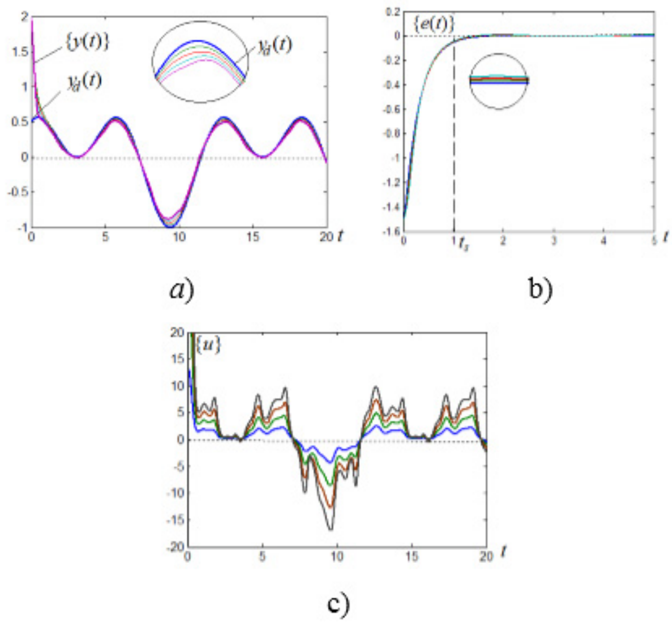

Figure 3: Dynamic features of the system that characterize robust properties

Figure 4 below shows a diagram of the implementation of the system on Simulink.

In order to be able to implement the system on Simulink, the “input-output” model (1) is rewritten in state variables.

Figure 5 shows the dynamic characteristics of the pendulum: a) - $y_d(t)$ reference trajectory, $y(t)$ adjustable output; b) - u control signal, obtained by the method (Levant, 2007), known in the literature as the “sliding mode of the n-th order” (Emelyanov et al., 1990).

As can be seen from Fig. 5a, the settling time is $t_s \approx 2.1$ *c*. This indicator significantly exceeds the required one $t_s = 1$ *c*. In addition, in the steady state, the control signal makes high-frequency oscillations with a high amplitude $u = \pm 10$. Such a regime leads to energy overexpenditure, and also limits the scope of application of the n-th order sliding regime (Utkin, 1992).

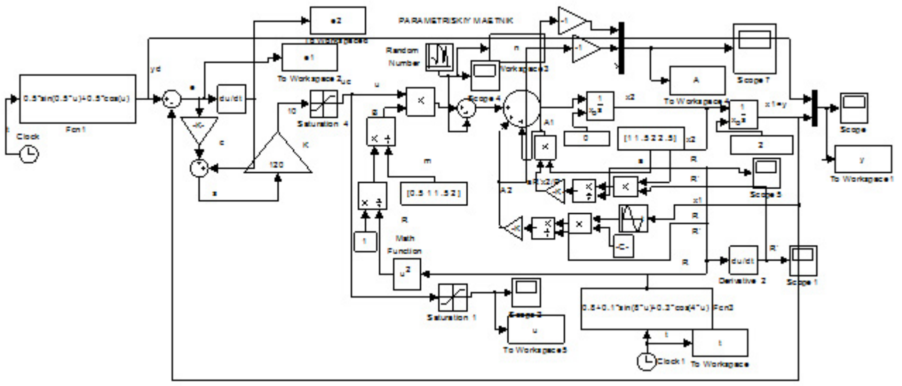

Figure 4: Implementation scheme $K\infty$ - robust Simulink systems

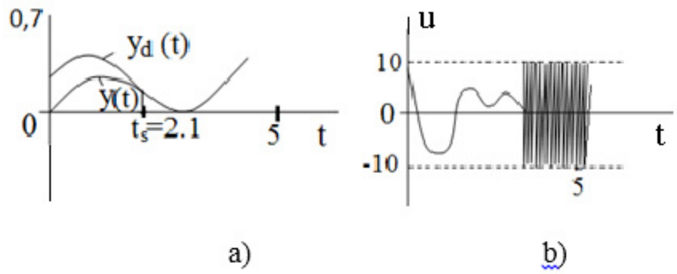

Figure 5: Dynamic characteristics of the system obtained on based on the sliding mode of the 2nd order

3 Conclusion

On the basis of theoretical and computer studies, the following main results have been obtained:

1. For the first time, the robust synthesis method with a high gain was applied to the control of the non-linear non-stationary object (peak pendulum) with parametric uncertainty;

2. The results of modeling the specific problem on *Matlab/Simulink* showed that the application of the method makes it possible to solve the mathematical difficulties typical for the robust synthesis problems with the wide class of uncertainties;

3. System-technical simplicity and simplicity of the control synthesis technique gives a wide opportunities for the practical application of robust systems with a large gain.

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